

International Journal of Modern Physics A
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Parton densities with the quark linear potential in the statistical approach

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Received Day Month Year

Revised Day Month Year

The statistical approach is used to calculate the parton distribution functions (PDFs) of the nucleon. At first it is assumed that the partons are free particles and the light-front kinematic variables are employed to extract the Bjorken x -dependence of the PDFs. These PDFs are used to evaluate the combinations of the sea quarks such as $\bar{d} - \bar{u}$. As our first attempt to improve the result, we make the statistical parameters to depend on Q^2 , using different values of Gottfried sum rule. The related results are indicating better behavior by accessing to the PDFs while they contain the Q^2 dependence parameters. As a further task and in order to have more improvement in the calculations, a linear potential is considered to describe the quark interactions. The solution of the related Dirac equation yields the Airy function and is considered as a wave function in spatial space. Using the fourier transformation the wave functions are obtained in momentum space. Based on the light-front kinematic variables and using a special method which we call it “k method”, these functions can be written in terms of the Bjorken x -variable. Following that the statistical features are accompanied with these functions. Considering an effective approach which is used in this article, we do not need to resort to any extra effects as were assumed in some articles to get a proper results for PDFs. The obtained results for $\bar{d} - \bar{u}$ and the $\frac{\bar{d}}{\bar{u}}$ ratio, using our effective approach, are in good agreement with the available experimental data and some theoretical results.

Keywords: statistical approach, light-front form, quark linear potential, Airy function

PACS numbers: 12.38.AW, 12.39.Ki, 12.40.Ee

1. Introduction

One of the important goals of particle physics is the description of nucleons from the first principles, i.e. determination of structure of nucleons (such as proton and

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neutron) in terms of quark and gluon degrees of freedom¹. To understand the reasons of breakdown of the efforts in this direction, first we should know the existing approaches to the problem and see where the difficulty lies. There are two fundamental different approaches to describe the structure of strongly interacting particles whose stable and natural states are called hadrons:

Parton model: this model was first introduced by Feynman² and assumes that nucleon is composed of point like constituents, named “partons”. After then it was clarified that partons are in fact consists of the quarks and gluons. This model was later on named constituent quark model, and has many phenomenological applications.

Quantum Chromodynamic (QCD): which is a SU(3) non-abelian quantum field theory. After the discovery of asymptotic freedom, this model became the accepted field theory to describe strong interactions.

Reconciling the above mentioned approaches has not been in a good manner successful up to now. The main reason for dilemma is that at first QCD is based mainly on perturbative calculations. Because of the presence of non-perturbative effects in hadronic physics, methods to incorporate non-perturbative effects should be invented. The front form of QCD³ appears to be the best tool for this purpose. The front form is also one of the basic tools for the statistical description of the parton model that is concerned in this paper, so it will be introduced briefly. On this base, the deep inelastic lepton proton scattering can be viewed as a sum of elastic lepton parton scattering, in which the incident lepton is scattered off a parton instantaneously and incoherently. In the statistical approach the proton is assumed as a equilibrium thermal system made up of free partons (quarks, anti-quarks and gluons) that they have energy and momentum due to their temperature⁴.

The organization of the paper is as follows : Introducing the statistical parton model is done in Sect. 2 and then some history and applications of front form are given there . Basic tools to enable us to employ the statistical approach are introduced in Sect.3. We present our results in Sect.4, where with the use of different values of Gottfried sum rule (S_G) at some energy scales, we achieve to the energy dependence of parton distributions in the employed statistical model. In continuation, relativistic quantum mechanical view is accompanied with the statistical approach. In this case quantum states of partons as fermions are obtained in Sect.5, using the solution of the Dirac equation under a linear potential which is proper to describe the typical interaction of the particles inside the nucleon. Following this strategy, the results for partons as a function of Bjorken x -variable can be calculated analytically which is done in Sect.6. We then resort to an effective approach to employ the quantum effect of the statistical approach in Sect.7 without resorting to use some extra effects which have been assumed in Ref.[5] . The results especially for $\bar{d} - \bar{u}$ and the $\frac{\bar{d}}{\bar{u}}$ ratio are in good agreement with the available experimental data and the theoretical results in Ref.[5]. We finally give our conclusions in Sect.8.

2. Parton model and the statistical approach

In the parton model approach, the nucleon structure functions are described in terms of parton distribution functions. As stated above, because of the non-perturbative effects it is not possible to calculate the PDF's completely from the perturbative part of the field theory, relating to the strong interactions. This is a sign that we are inevitable to investigate other models which can be described properly both the perturbative and non-perturbative parts of QCD. Since quarks and gluons are confined in a volume of nucleon size, it is expected that statistical properties are important in determining parton distribution functions^{4,6,7}. So the statistical parton model is introduced to incorporate the effects which are related to the properties of partons. In the primarily statistical parton model it is assumed that the nucleon is a gas of non-interacting valence quarks, sea quarks, anti-quarks and gluons with a thermal equilibrium.

It is explained that the proper language for high energy region is light-front dynamics. Light front dynamics is related to infinite momentum frame (IMF). It is not a straight forward job to transit from ordinary instant-form approximation to front form dynamics. Instant-form approximation is suitable for non relativistic dynamics and assumes that the system is prepared in an instant of time in its rest frame, and evolves to later time. As is apparent it is not suitable for relativistic dynamics. The front form is the suitable language and because of its importance for statistical parton model, we insist on using it. To describe the front form, let us have a review on the historical progress of the parton model.

The parton model was first initiated by Feynman². Then the scaling behavior of partons in deep inelastic scattering was indicated by Bjorken⁸ in the limit of infinite momentum frame. Dirac⁹ was the first who introduced the concept of light-front form for relativistic dynamics, which he called "front form". Another important application of light-front dynamics is in the deep inelastic scattering processes. Later on Weinberg developed a new formulation for the perturbative field theory in the IMF¹⁰ and it was realized that IMF and light front dynamics are equivalent and scaling behavior can be understood more easily in the light-front form¹¹.

In another approach to deal with the parton model, impulse approximation is used in deep inelastic lepton-nucleon scattering in perturbative field theory in the IMF¹². Impulse approximation can be used when the time of interaction between the projectile and the constituent of the target is much smaller than the life time of the target as seen by projectile¹². In hadronic physics the light transit time is comparable to the time scale governing internal motions so it might seem that only a fuzzy picture of instantaneous state of the hadron can be obtained¹³. The solution is to study the hadron in a reference frame with the speed near that of light. In this case the time dilation effect slows the internal motions so that impulse approximation can be used.

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3. Basic concepts of the statistical model

By the assumption that nucleon is a system of partons in thermal equilibrium, the mean number of partons (in nucleon rest frame) is given by ^{14,15}:

$$\bar{N}_f = \int f(k^0) d^3k, \quad (1)$$

where $f(k^0)$ is a distribution function (Fermi-Dirac or Bose-Einstein distribution):

$$f(k^0) = \frac{g_f V}{(2\pi)^3} \frac{1}{e^{\frac{k^0 - \mu_f}{T}} \pm 1}. \quad (2)$$

In this equation the plus sign is referring to fermions (quark, anti-quark) and the minus sign is for the bosons (gluon). The g_f is the degree of color-spin degeneracy (6 for quarks and anti-quarks, 16 for gluon). Chemical potential is representing by μ_f . As is known from thermodynamic the sign for anti-particle is opposite to that of particle ($\mu_{\bar{q}} = -\mu_q$), and for massless bosons the chemical potential is zero ($\mu_g = 0$).

The four vector of energy-momentum in Eq.(1) is defined by:

$$\vec{k} = (k^1, k^2, k^3), \quad k^0 = \sqrt{\vec{k}^2 + m_f^2}, \quad (3)$$

where k^0 is the energy, \vec{k} is 3-momentum and m_f is mass of parton. By imposing the on-shell condition on Eq.(1), we will get:

$$\bar{N}_f = \int f(k^0) \delta(k^0 - \sqrt{(k^3)^2 + k_\perp^2 + m_f^2}) dk^0 dk^3 d^2k_\perp, \quad (4)$$

To transform them to light-front kinematic variables in the nucleon rest frame, the following transformations are used:

$$k^+ = k^0 + k^3, \quad k_\perp = (k^1, k^2), \quad k^- = k^0 - k^3, \quad k^+ = P^+ x = Mx, \quad (5)$$

where x is the light-front momentum fraction of nucleon carried by parton and M is the mass of nucleon. Using the light-front variables the delta function and integration measure changes to:

$$\begin{aligned} \delta(k^0 - \sqrt{(k^3)^2 + k_\perp^2 + m_f^2}) &= 2k^0 \theta(k^0) \delta(k^2 - m_f^2) \\ &= [1 + \frac{k_\perp^2 + m_f^2}{Mx^2}] \theta(x) \delta(k^- - \frac{k_\perp^2 + m_f^2}{Mx}), \end{aligned} \quad (6)$$

$$dk^0 dk^3 d^2k_\perp = \frac{1}{2} M dk^- dx d^2k_\perp. \quad (7)$$

Therefore Eq.(4) will appear as:

$$\bar{N}_f = \int f(x, k_\perp) dx d^2k_\perp, \quad (8)$$

where $f(x, k_\perp)$ is resulted by substituting the Eq.(7) in Eq.(4) and integrating it with respect to k^- :

$$f(x, k_\perp) = \frac{g_f MV}{2(2\pi)^3} \frac{1}{\exp(\frac{1/2(Mx + \frac{K_\perp^2 + m_f^2}{Mx}) - \mu_f}{T}) \pm 1} [1 + \frac{K_\perp^2 + m_f^2}{(Mx)^2}] \theta(x), \quad (9)$$

Integrating Eq.(9) with respect to k_\perp (with the assumption of being isotropic in transverse plane), will yield us:

$$f(x) = \frac{g_f MTV}{8\pi^2} \{ (Mx + \frac{m_f^2}{Mx}) \text{Ln}[1 \pm \text{Exp}(-\frac{1/2(Mx + \frac{m_f^2}{Mx}) - \mu_f}{T})] - 2T \text{Li}_2(\mp \text{Exp}(-\frac{1/2(Mx + \frac{m_f^2}{Mx}) - \mu_f}{T})) \}, \quad (10)$$

where upper sign is denoting to fermions, negative sign to bosons and Li_2 is the poly-logarithm function. Eq.(10) is representing the parton distribution function whose free parameters will be determined, using the related constraints. The statistical parameters of the proton, will be obtained if the following sum rules are fulfilled:

$$u_v = \int_0^1 [u(x) - \bar{u}(x)] dx = 2, \quad (11)$$

$$d_v = \int_0^1 [d(x) - \bar{d}(x)] dx = 1, \quad (12)$$

$$\int_0^1 x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + g(x)] dx = 1. \quad (13)$$

The free parameters of PDF in Eq.(10) are T , V , μ_u and μ_d . As is proposed in ⁴, the value of T is known and the other parameters will be determined, using Eqs.(11,12,13). To obtain the four unknown parameters in Eq.(10) we need to an extra constrain in addition to the existed sum rules for partons. This extra constrain is related to a reasonable value for S_G in correspond to the available experimental data. Therefore there are four parameters which need be obtained while the four existed equations should be solved simultaneously. The S_G is given by:

$$S_G = \int_0^1 \frac{F_2^p - F_2^n}{x} dx = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx, \quad (14)$$

The experimental result for Gottfried sum rule is $S_G = 0.235 \pm 0.026$ ¹⁶. The authors in ⁴ have found the Gottfried sum rule at value $T=47 \text{ MeV}$, equals to $S_G = 0.236$ which agrees well with the experimental data. The other three parameter values are: $V = 1.2 \times 10^{-5} \text{ MeV}^{-3}$, $\mu_u = 64 \text{ MeV}$, $\mu_d = 36 \text{ MeV}$.

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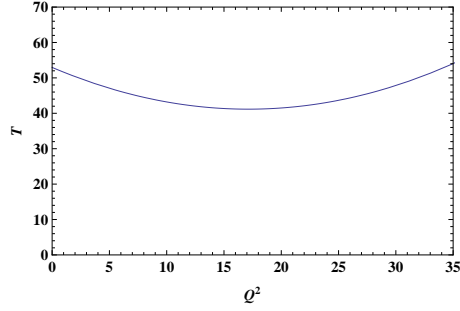


Fig. 1. Temperature parameter, T , versus Q^2 .

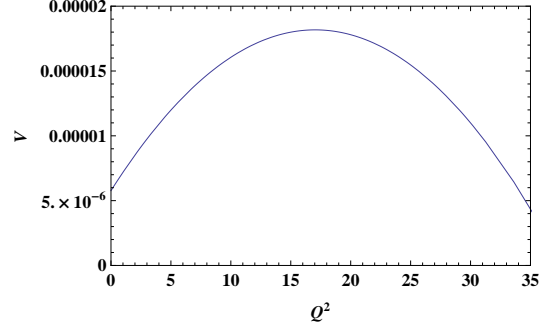


Fig. 2. Volume parameter, V , versus Q^2 .

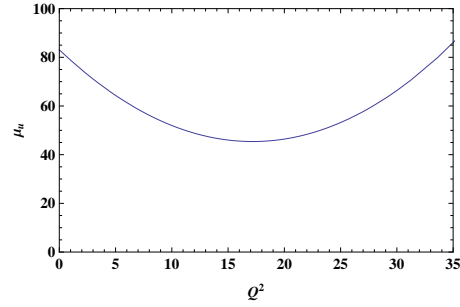


Fig. 3. Chemical potential for up quark, μ_u , versus Q^2 .

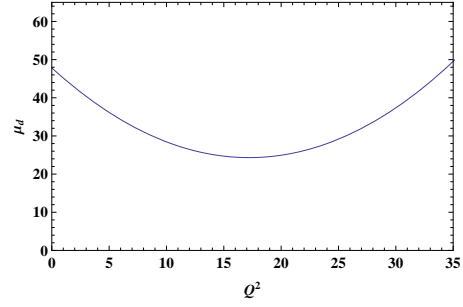


Fig. 4. Chemical potential for down quark, μ_d , versus Q^2 .

4. Energy dependence of parameters

With the x -dependent distribution function which we access it according to Eq.(10) and solving the set of Eqs.(11-14) simultaneously, the four parameters of statistical parton model, T , V , μ_u and μ_d can be computed at desired energy scales which can be related to the S_G values at some available energy scales, quoted in [17, 18]. In Ref.[18] the required plot of S_G against Q^2 has been plotted.

Therefore we achieve to the temperature, volume and chemical potentials at different energy scales. Then we are able to fit a polynomial function to the data points, related to different values of the statistical parameters at different energy scales which are corresponded to the various energy scales of S_G in [17]. We can then obtain energy dependence of the parameters in the used statistical model (T , V , μ_u , μ_d). The fitted polynomial function for the concerned parameters as a function of Q^2 is chosen according to the following general form:

$$g(Q) = pQ^2 + qQ + r. \quad (15)$$

The p , q and r are the parameters which are determined by a fit and can be assigned them the values as listed in Table.2. In Fig. 1-4 we plot the V , T , μ_u and μ_d parameters as a function of Q , based on the Eq.(15). In Fig. 5 we plot the $\bar{d} - \bar{u}$

Table 1. The numerical values of parameters in Eq.(15).

$g(x)$	V	T	μ_u	μ_d
p	-4.27204×10^{-8}	0.0402465	0.127655	0.0794809
q	1.45577×10^{-6}	-1.37801	-4.38716	-2.73393
r	5.77185×10^{-6}	52.9615	83.094	47.8348

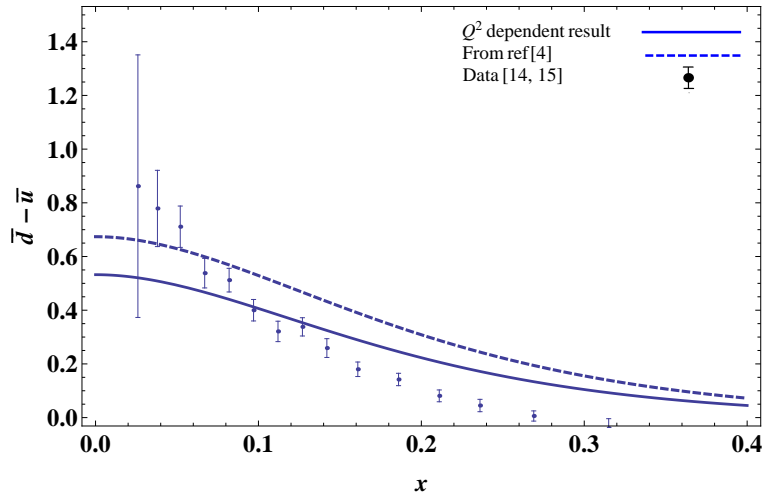


Fig. 5. The difference of $\bar{d}-\bar{u}$, resulted from Eq.(10) (dotted curve). The solid curve is resulted from the the Q^2 dependence of the statistical parameters. Experimental data is quoted from ^{19,20}

at $Q^2 = 54 \text{ GeV}^2$ when we use Eq.(10) which does not contain Q^2 -dependence. A comparison with this quantity when we assume a Q^2 -dependence for related parameters in Eq.(10), based on Eq.(15) has also been done. As can be seen by making a Q^2 -dependence for the statistical parameters, we achieve to a little bit better agreement for $\bar{d}-\bar{u}$ in comparison with the available experimental data ^{19,20}.

5. Confining potential and the statistical approach

To obtain the flavor asymmetry in the sea of the nucleon, given by $\frac{\bar{u}}{\bar{d}}$ and $\bar{d}-\bar{u}$, a phenomenological statistical model presented in ^{21,5} is followed with linear confined quarks. In this model the temperature parameter is given by the S_G violation and the chemical potentials by the net number of u and d quarks in the nucleon. To describe the nucleon, two different chemical potentials are required in which one fixes the net number of u quarks and the other one the net number of d quarks. The models studied in ^{22,23} inspire the given approach in ²¹. Here the quark energies are not assuming to have continuum levels as in ^{22,23}. In contrast, a Dirac confining potential ²⁴ were considered to generate the single-particle spectrum in which the

given quark levels obey the Fermi statistics. In this case for each quark flavor λ_q , the strength of the confining potential λ_q , is fitted through the hadron masses where $m_u = m_d = 0$ and it was assumed $\lambda_u = \lambda_q = \lambda$.

It is not an easy task to develop a quark model with confining potential to incorporate the x -dependence of quark distribution functions. By assuming the non-perturbative properties of the hadron wave function, the recent approach to incorporate the x -dependence of the quark distribution functions is obtained as suggested in ²⁵. In the present statistical quark model, using a dynamical input, i.e., the relativistic quark confining potential, we can achieve to the x -dependence of the probability functions and the related observable. To extract the x -dependence of the quark amplitudes, the momentum representation of the quark eigenstates, related to the Dirac Hamiltonian is written with respect to the light-cone momenta. The single-particle nature of the model makes simple this transformation.

All the individual quarks of the system including the valence and sea quarks in the present statistical quark model are confined by a central effective interaction. The scalar and vector components of the related potential with strength λ , is given by ²⁴:

$$V(r) = (1 + \beta) \frac{\lambda r}{2}. \quad (16)$$

For the u and d quarks, the strength λ of this potential is identical. Considering the chosen potential in Eq.(16), an equation similar to the Schrodinger equation is resulted from the coupled Dirac equation which can be solved by using the conventional methods of non-relativistic dynamics. The related Dirac equation which should be solved is given by

$$[\vec{\alpha} \cdot \vec{p} + \beta m + V(r)] \psi_i(\vec{r}) = \varepsilon_i \psi_i(\vec{r}). \quad (17)$$

The β and $\vec{\alpha}$ are the fourth usual 4×4 Dirac matrix. They can be written in terms of the 2×2 Pauli matrices. Using $\psi_i(\vec{r})$ which is given by

$$\psi_i(\vec{r}) = \begin{pmatrix} 1 \\ \vec{\sigma} \cdot \vec{p} / (m + \varepsilon_i) \end{pmatrix} \varphi_i(\vec{r}), \quad (18)$$

the final coupled equations can be transformed to a single second order differential equation such that:

$$[p^2 + (m + \varepsilon_i)(m + \lambda r - \varepsilon_i)] \varphi_i = 0. \quad (19)$$

Using partial wave expansion, this equation can be solved for the radial part and in the case of the s wave ($\ell = 0$), where $j^p = (1/2)^+$, the radial part of φ_i can be written in terms of the Airy function (Ai):

$$\varphi_i(r) = \sqrt{\frac{k_i}{4\pi}} \frac{Ai(k_i r + a_i)}{r \left[\frac{dAi(x)}{dx} \right]_{x=a_i}}. \quad (20)$$

The a_i is the related i th root of $A_i(x)$, $k_i = \sqrt[3]{\lambda(m + \varepsilon_i)}$, m is denoting the current quark mass, finally ε_i are representing the energy levels and are given by

$$\varepsilon_i = m - \frac{\lambda}{k_i} a_i . \quad (21)$$

The energies for the u and d quarks with $m = 0$ are given by

$$\varepsilon_i = \sqrt{\lambda}(-a_i)^{3/4} . \quad (22)$$

6. Statistical approach and the quark model

Using the relativistic linear confining potential, the statistical quark model can be investigated to yield us the nucleon structure function. Via this model²⁶, the energy levels for the quarks, ε_i , can be determined. In this model the nucleon involves three valence quarks and the sea quarks. Here the contribution of gluon fields is small and can be neglected^{27,28}.

For the quarks in the present statistical quark model, the Fermi–Dirac distribution is assumed. For a quark system, with energy levels ε_i and temperature T , the probability density for a quark system is given by

$$\rho_q(\vec{r}) = \sum_i g_i \psi_i^\dagger(\vec{r}) \psi_i(\vec{r}) \frac{1}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)} . \quad (23)$$

The $|\psi_i(\vec{r})|^2$ is representing the density probability for each state which is normalized to 1 and the g_i gives the level degeneracy.

The current quark masses given by $m_u = m_d = 0$ are considered here which are in fact contains the light quarks. Using a confining potential model in the Dirac equation, the energies for the u and d quarks are taken to be equal. With this assumption the normalization for the proton is as follows:

$$\begin{aligned} & \int [\rho_q(r) - \bar{\rho}_q(r)] d^3r \\ &= \sum_i g_i \left[\frac{1}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)} - \frac{1}{1 + \exp\left(\frac{\varepsilon_i + \mu_q}{T}\right)} \right] = \begin{cases} 1 & \text{for } q = d \\ 2 & \text{for } q = u \end{cases} . \end{aligned} \quad (24)$$

In order to calculate the nucleon structure function, it is needed to write the quark wave function in momentum space, taking the Fourier transform:

$$\Phi_i(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int \exp(-i\vec{p} \cdot \vec{r}) \psi_i(r) d^3r . \quad (25)$$

Considering the probability density as $\varrho = \Phi_i^\dagger(p) \Phi_i(p)$, the quark distribution in the Bjorken- x space is obtained using two different methods:

In the first method which we call it “k method” the required distribution is obtained by the relation^{29,30}:

$$q(x) = 2\pi M_t \int_{k_{min}}^{\infty} \varrho p dp , \quad (26)$$

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where

$$k_{min} = \frac{xM_t + \varepsilon_i}{2} - \frac{m^2}{2(xM_t + \varepsilon_i)} .$$

In this equation M_t and m are denoting the proton and u, d quark masses respectively. The energy level of ground state is given by ε_i .

In the second method which we call it “null-plane method”, to render the parton distribution in the Bjorken x -space, the quark distribution function is obtained by the relation ¹⁴:

$$q(x) = \int q(x, \vec{p}_\perp) d^2 p_\perp , \quad (27)$$

where $q(x, \vec{p}_\perp)$ satisfies the following relation:

$$\int \varrho \delta(p^0 - \sqrt{(p^3)^2 + (\vec{p}_\perp)^2 + m^2}) dp^0 dp^3 d^2 p_\perp = \int q(x, \vec{p}_\perp) d^2 p_\perp dx . \quad (28)$$

Note that when doing the integration in Eq.(28), the on-shell condition $p^0 = \sqrt{\vec{p}^2 + m^2}$ is needed, where $p^0, \vec{p} = (p^1, p^2, p^3)$ and m are the energy, 3-momentum and mass of the quark respectively. We should emphasize that the “null-plane method” which was also used in Refs.[5, 21] was the main motivation to lead us to follow the calculations based on the first method.

In brief as was dealt with in Sect. 3, using the null plane variables, we will have

$$\begin{aligned} p^+ &= xP^+, \quad P^+ = M_t(t = p, n) \\ p_z &= p^+ - \varepsilon_i = M_t \left(x - \frac{\varepsilon_i}{M_t} \right) , \end{aligned} \quad (29)$$

where x is the momentum fraction of the nucleon carried by the quark, M_t is the nucleon thermal mass at a given temperature T . Therefore we redefine the wave function in Eq.(25) as

$$\Phi_i(\vec{p}) = \Phi_i(x, p_\perp) . \quad (30)$$

By integrating Eq.(23) over p_\perp where $\psi(\vec{r})$ is replaced by $\Phi_i(x, p_\perp)$ we obtain the quark structure function for each flavor q as in the following:

$$q_T(x) = \sum_i \int d^2 p_\perp \frac{\Phi_i^\dagger \Phi_i}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)} . \quad (31)$$

In fact for Φ_i in Eq.(30) we have:

$$\Phi_i = \Phi_i \left(M_t \left(x - \frac{\varepsilon_i}{M_t} \right), p_\perp \right) . \quad (32)$$

In Eq.(31), $q_T(x)$ describes the probability that a quark with flavor q has a fraction x of the total momentum of the nucleon, assuming a temperature T . For the corresponding anti-quark distribution, $\bar{q}_T(x)$, we have to replace μ_q by $\mu_{\bar{q}}$ in Eq.(31).

7. An effective approach to the linear confining potential

In a standard procedure, we should take massless quarks and equal potential strength for different flavor quarks and then to do a sum over different energy levels as in Eq.(31) to achieve the quark distributions as a function of Bjorken- x variable. What we do in this section is different with respect to what was assumed in previous section. Here we assign different masses to u and d quarks so as: $m_u=187 \text{ MeV}$, $m_d=196 \text{ MeV}$ as current quark masses. The strength of linear potential which is denoted by λ in Eq.(16) is assumed identical for u and d quarks in which we take into account $\lambda=239 \text{ MeV}$ in correspond to what was quoted in ²¹. We assume just one energy level effectively and due to the different quark mass, the amount of energy level is not identical for different quark flavors so as we take $\varepsilon_u=290 \text{ MeV}$ and $\varepsilon_d=225 \text{ MeV}$ for u and d quarks respectively. This will lead us to the different values for k_i in Eq.(21). In our calculations the second root of Airy function is used, $a_2=-4.08798$, which is more appropriate for taken effective approach in this section.

As was pointed out in previous section to convert the quark wave function from momentum space to Bjorken- x space, we employ the second method which was called “k method”. The integral in Eq.(25) is a triple integral. The integral over azimuthal variable can be done strictly due to cylindrical symmetry. The integration over polar variable from $\theta=0$ to $\theta=\pi$ can be easily calculated. The final integration over radial variable, r , can also be done analytically. By replacing the final result for $\Phi_i(p)$ in Eq.(26), we can obtain the quark distributions in terms of the x -variable. If one wishes to do the integration in Eq.(26) analytically, it will be very hard so we prefer to do this integration numerically. For this propose, we need to make a data table which contains two columns. First column includes the different values of p variable from $p=-a$ to $p=a$ where a is a large number. The chosen interval for p is due to this fact that the $\Phi_i(p)$ is an even function with respect to p . The second column is containing the numerical values of $p\rho$ ($\rho = \Phi_i^\dagger(p)\Phi_i(p)$), considering the different values of p -values in the first column. By considering the shape of the $p\rho$ combination with respect p which can be obtained by a list plot of extracted data, a proper fitting function for the concerned quantity can be conjectured. We take the following function for the fit:

$$f(p) = a p e^{-b p^2} + c p^3 e^{-d p^4} + h p e^{-g p^6}, \quad (33)$$

where a, b, c, d, h and g are the fitting parameters. By substituting the fitted parameters for u and d quark respectively in Eq.(33) and then back the result into the Eq.(26), the related quark distribution in x -space will be obtained. The numerical value for M_t in this equation is taken to equal to $M_t=985 \text{ MeV}$. In this stage of calculations, we can normalize the bare quark distributions for u and d quarks to 2 and 1 as are required.

The next step is to impose this result on the employed statistical effect by multiplying it with the related factor in Eq.(31), that is:

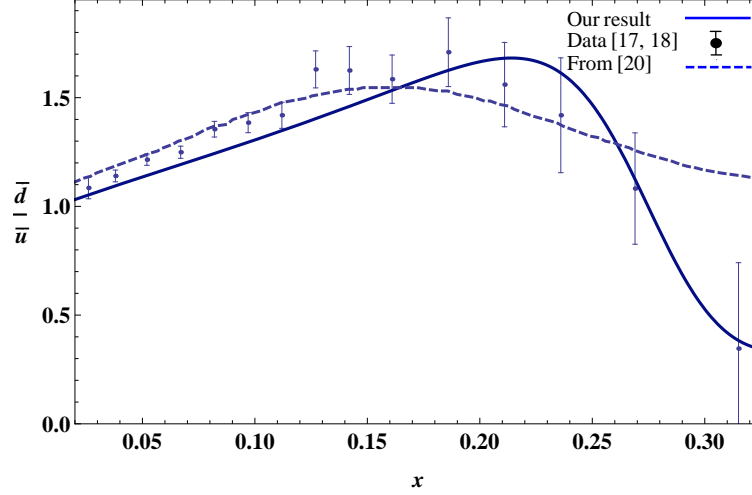


Fig. 6. The ratio of $\frac{\bar{d}}{\bar{u}}$, resulted from Eq.(31)(solid curve) in an effective approach. Experimental data is quoted from ^{19,20}. Comparison with the result from [5] (dash-dotted curve) has also been done.

$$\frac{1}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)}. \quad (34)$$

Then by putting the final results in the sum rules, given by Eqs.(11,12) and Eq.(14), we can obtain the unknown parameters μ_u , μ_d and T by solving simultaneously the related set of equations. The numerical values for these parameters which we obtain, are : $\mu_u=908.42 \text{ MeV}$, $\mu_d=720.094 \text{ MeV}$ and $T=219.56$. Since the calculations are done effectively, the numerical values for temperature and chemical potentials would be different with respect to what are existed in the usual statistical model. The essential point which we should take into account is that the numerical behavior of these quantities is like the one which is expecting from the usual statistical model.

By substituting these numerical values in Eq.(31) and choosing the proper sign and values for the chemical potential, we can achieve to \bar{u} and \bar{d} distributions. The results for $\frac{\bar{d}}{\bar{u}}$ and $\bar{d} - \bar{u}$ at $Q^2=54 \text{ GeV}^2$ are depicted in Fig.6 and Fig.7. The comparison with the available experimental data ^{19,20} and the result from Ref.[5] has also been done there.

As we mentioned at the beginning of this section we use effectively the confining linear potential to indicate that the model is working well. In fact we intend to show that this model has inherently this ability to give us an acceptable result for quark distributions. We get these results by choosing proper numerical values for quantities like m_u , m_d , ... as we refer them at the first part of this section. Using these values as we explained before, we extract the $\frac{\bar{d}}{\bar{u}}$ and $\bar{d} - \bar{u}$ in a good agreement with the available experimental data and even in a better agreement (more and

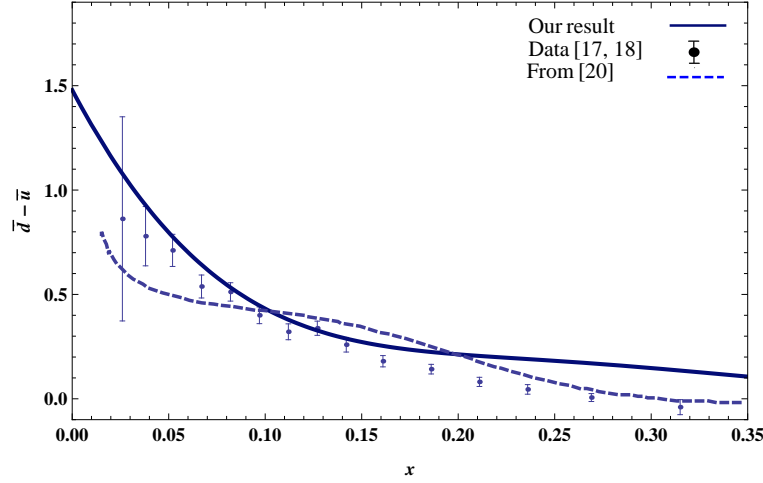


Fig. 7. The difference of $\bar{d}-\bar{u}$, resulted from Eq.(31)(solid curve) in an effective approach. Experimental data is quoted from Refs.[19, 20]. Comparison with the result from [5] (dash-dotted curve) has also been done.

less) with the results from Ref.[5]. Once again we emphasize that our approach is an effective approach and is not in complete correspondence to what described in previous section.

As can be seen in ⁵, just using the linear confining potential of Sect.6 with the assumptions which existed there, will not give us the proper results for the $\frac{\bar{d}}{u}$ and $\bar{d} - \bar{u}$. In fact to achieve to the proper results, following the strategy of Sect.6, we should add some extra effects to the calculations, such as mass shift, instanton effect and quark substructure like gluonic and pionic effects as were used in ⁵. Considering these extra effects will yield the proper results for the concerned quantities. But we show in this section that we can obtain the proper results without using these extra effects, just by following an effective approach which we describe here, using chosen numerical values for the required quantities.

8. Conclusions

Here we had a review on the statistical approach based on the Ma *et al.*'s article⁴ to obtain the parton densities inside the nucleon. In further step we did dependence the related statistical parameters on Q^2 by using the different values of S_G at different values of Q^2 in correspond to neural method in Ref.[17]. In this case the result for $\bar{d}-\bar{u}$ at $Q^2=54 \text{ GeV}^2$ would be in better agreement with the available experimental data ^{19,20} as can be seen in Fig.5.

So far the partons were assumed as free particles. In real case the interaction between partons should be considered which has been done in Sect.6 by taking into account a linear confining potential between the quarks. Employing this interaction would improve the results for the extracted parton densities with respect to the

case where the partons were taken as free particles. For this propose, at first the wave function of quarks were obtained in spatial coordinates, using the solution of the Dirac equation under the linear confining potential. In continuation, the quark wave function should be converted to momentum space which could be done by a Fourier transformation. Later on this wave function would be appeared in Bjorken x -space by two methods which were called “null-plane method” and “k method” as were described in Sect.6. We used in this article the “k method” method to obtain the quark distributions in x -space.

According to the descriptions of Sect.6, the real quark distribution would be obtained by doing a summation over different energy levels of quarks in confining potential while massless quarks were assumed. In the effective approach which we took into account in Sect.7, we assumed massive quarks with one energy level which were different for u and d quarks. What we got in this approach were acceptable and in a good agreement with $\bar{d}-\bar{u}$ and the ratio $\frac{\bar{d}}{\bar{u}}$ at $Q^2 = 54 \text{ GeV}^2$ in comparison with the available experimental data and the obtained result from ⁵. This showed that the linear confining potential contains alone and inherently this ability to produce proper results for parton densities without resorting to some extra effects like mass shift and etc. as were done in Ref.[5].

Considering other confining potential like M.I.T bag model or squared radial potential would also be interesting to yield us the parton densities in x -space. We hope to do in this connection some reports in our further research job. Extending the calculations to the polarized case would be as well an attractive subject to follow it as our new scientific task.

Appendix A.

We indicate here the details of numerical calculations, based on Eq.(26) which lead us to parton densities, using linear confining potential.

Employing the numerical values for quark binding energies and the other numerical values for the required quantities as are indicated in the article, the result of integration in Eq.(25) with respect to p variable for the u quark is as follows:

$$\begin{aligned} \Phi_u(p) = & \frac{1}{p^3} 0.00137543 i e^{9.95192 i p - 4.80931 i p^3} (-21.7656 i \sqrt[3]{-ip^3} p - 21.7656 (-i p^3)^{2/3} \\ & + 16.0736 i \sqrt[3]{-ip^3} p \Gamma(\frac{2}{3}, -4.80931 i p^3) + 8.12471 (-ip^3)^{2/3} \Gamma(\frac{1}{3}, -4.80931 i p^3) + 4\sqrt{3}\pi p^2) \\ & - \frac{1}{p^3} 0.00137543 i e^{4.80931 i p^3 - 9.95192 i p} (21.7656 i \sqrt[3]{ip^3} p - 21.7656 (i p^3)^{2/3} \\ & - 16.0736 i \sqrt[3]{ip^3} p \Gamma(\frac{2}{3}, 4.80931 i p^3) + 8.12471 (i p^3)^{2/3} \Gamma(\frac{1}{3}, 4.80931 i p^3) + 4\sqrt{3}\pi p^2) . \end{aligned} \quad (\text{A.1})$$

For the d quark we will have:

Table 2. The amounts of $p\rho_i = p\Phi_i^\dagger(p)\Phi_i(p)$ versus different values of p .

p	$p\Phi_u^\dagger(p)\Phi_u(p)$	$p\Phi_d^\dagger(p)\Phi_d(p)$
-0.2	-0.00799423	-0.00429547
-0.19	-0.0104922	-0.00645628
-0.18	-0.0131948	-0.00899122
-0.17	-0.0160138	-0.0118213
-0.16	-0.0188475	-0.0148434
-0.15	-0.0215853	-0.017934
-0.14	-0.0241117	-0.0209543
-0.13	-0.0263111	-0.0237567
-0.12	-0.0280731	-0.0261918
-0.11	-0.0292978	-0.0281166
-0.10	-0.0299005	-0.0294021
-0.09	-0.0298161	-0.029941
-0.08	-0.0290029	-0.029654
-0.07	-0.0274451	-0.0284955
-0.06	-0.0251548	-0.0264573
-0.05	-0.0221718	-0.0235702
-0.04	-0.018563	-0.0199035
-0.03	-0.0144202	-0.0155633
-0.02	-0.00985654	-0.0106873
-0.01	-0.00500241	-0.00543901
0.00	0.00000004	0.000000003
0.01	0.00500241	0.00543901
0.02	0.00985654	0.0106873
0.03	0.0144202	0.0155633
0.04	0.018563	0.0199035
0.05	0.0221718	0.0235702
0.06	0.0251548	0.0264573
0.07	0.0274451	0.0284955
0.08	0.0290029	0.029654
0.09	0.0298161	0.029941
0.10	0.0299005	0.0294021
0.11	0.0292978	0.0281166
0.12	0.0280731	0.0261918
0.13	0.0263111	0.0237567
0.14	0.0241117	0.0209543
0.15	0.0215853	0.017934
0.16	0.0188475	0.0148434
0.17	0.0160138	0.0118213
0.18	0.0131948	0.00899122
0.19	0.0104922	0.00645628
0.20	0.00799423	0.00429547

$$\begin{aligned}
\Phi_d(p) = & \frac{1}{p^3} 0.00131846 i e^{10.8304ip-6.19867 i p^3} (-21.7656 i \sqrt[3]{-ip^3} p - 21.7656 (-ip^3)^{2/3} \\
& + 16.0736 i \sqrt[3]{-ip^3} p \Gamma(\frac{2}{3}, -6.19867 i p^3) + 8.12471 (-i p^3)^{2/3} \Gamma(\frac{1}{3}, -6.19867 i p^3) \\
& + 4\sqrt{3} \pi p^2) - \frac{1}{p^3} 0.00131846 i e^{6.19867 i p^3-10.8304 i p} (21.7656 i \sqrt[3]{ip^3} p - 21.7656 (i p^3)^{2/3} \\
& - 16.0736 i \sqrt[3]{ip^3} p \Gamma(\frac{2}{3}, 6.19867 i p^3) + 8.12471 (i p^3)^{2/3} \Gamma(\frac{1}{3}, 6.19867 i p^3) + 4 \sqrt{3} \pi p^2) .
\end{aligned} \tag{A.2}$$

In order to calculate the integration in Eq.(26), we resort to a numerical method. For this propose we need first to make a table of data, inclining two columns: one the p variable in a symmetrical interval, say for instance from -2 to +2 and the other one is the integrated quantity in Eq.(26). In the obtained table, the chosen step is 0.1. In practice, the amount of the step can be chosen less than the assumed step. To avoid of presenting two separate tables, we merge in related table the data for u and d quarks.

Considering the shape of the data, the best function to fit the data for u quark, is as following:

$$f_u(p) = a_u p e^{-b_u p^2} + c_u p^3 e^{-d_u p^4} + h_u p e^{-g_u p^6}, \quad (\text{A.3})$$

Fitting the above equation to the related data will lead us to the following numerical values for the unknown parameters of the equation:

$$\begin{aligned} a_u &\rightarrow 0.502886, & b_u &\rightarrow 46.3325, & c_u &\rightarrow -1.77947, \\ d_u &\rightarrow 379.021, & h_u &\rightarrow -2.88708, & g_u &\rightarrow 122054. \end{aligned} \quad (\text{A.4})$$

For the d quark, the fitted equation is like the u one, so as:

$$f_d(p) = a_d p e^{-b_d p^2} + c_d p^3 e^{-d_d p^4} + h_d p e^{-g_d p^6}, \quad (\text{A.5})$$

The numerical results for the unknown parameters are as following:

$$\begin{aligned} a_d &\rightarrow 0.547355, & b_d &\rightarrow 55.0649, & c_d &\rightarrow -2.22766, \\ d_d &\rightarrow 516.719, & h_d &\rightarrow -4.47365, & g_d &\rightarrow 148614. \end{aligned} \quad (\text{A.6})$$

The desired form of Eq.(26) for u quark is appeared as:

$$u(x) = N_u 2\pi M_t \int_{k_{min_u}}^{\infty} \varrho_u p dp, \quad (\text{A.7})$$

where

$$k_{min_u} = \frac{xM_t + \varepsilon_u}{2} - \frac{m_u^2}{2(xM_t + \varepsilon_u)} \text{ and } \varrho_u = \Phi_u^\dagger(p)\Phi_u(p).$$

In this equation, N_u is a normalization constant to fulfil the required sum rule for the u quark. Substituting the final form of Eq.(A.3) which is representing $\varrho_u p$, in Eq.(A.7) will yield us the u quark distribution in the Bjorken x space:

$$\begin{aligned}
 u(x) = & 8515.86(-1.779(0.0006595 - \frac{1}{(0.985x + 0.29)^4} e^{-\frac{20.991(x+0.104569)^4(x+0.484264)^4}{(0.985x+0.29)^4}} \\
 & (0.000620901 e^{\frac{20.991(x+0.104569)^4(x+0.484264)^4}{(0.985x+0.29)^4}} - 0.000620901)(x + 0.294374)(x + 0.294458) \\
 & (x^2 + 0.588832x + 0.0866809)) + (0.502886(e^{-\frac{10.9036(x+0.104569)^2(x+0.484264)^2}{(0.985x+0.29)^2}} \\
 & ((0.0107915x + 0.00635442)x + 0.000935422) - 1.787959984515764 \times 10^{-18} \\
 & (x + 0.25)^2)/(1.x + 0.294416)^2 - 0.00000394234 e^{-\frac{1590.77(x+0.104569)^6(x+0.484264)^6}{(0.985x+0.29)^6}} \\
 & - 6.113633443709904 \times 10^{-22}) \tag{A.8}
 \end{aligned}$$

Similarly for d quark, the required substitution should be done in following integration:

$$d(x) = N_d 2\pi M_t \int_{k_{min_d}}^{\infty} \varrho_d p dp, \tag{A.9}$$

where

$$k_{min_d} = \frac{xM_t + \varepsilon_d}{2} - \frac{m_d^2}{2(xM_t + \varepsilon_d)} \text{ and } \varrho_d = \Phi_d^\dagger(p)\Phi_d(p).$$

The result of integration in Eq.(A.9) for d quark in x space, would be as in following. As before the N_d is the required normalization constant.

$$\begin{aligned}
 d(x) = & 2573.23(-2.2276(0.00048382 - \frac{1}{(0.985x + 0.225)^4} e^{-\frac{28.617(x+0.0294416)^4(x+0.427411)^4}{(0.985x+0.225)^4}} \\
 & (0.000455439 e^{\frac{28.617(x+0.0294416)^4(x+0.427411)^4}{(0.985x+0.225)^4}} - 0.000455439)(x + 0.228385)(x + 0.228468) \\
 & (x^2 + 0.456853x + 0.0521786)) + (0.547355(3.575919969031528 \times 10^{-18} \\
 & (x + 0.125)(x + 0.375) + e^{-\frac{12.9586(x+0.0294416)^2(x+0.427411)^2}{(0.985x+0.225)^2}} \\
 & ((0.0090802x + 0.00414831)x + 0.000473792)))/(1.x + 0.228426)^2 - 0.000005017 \\
 & e^{-\frac{1936.93(x+0.0294416)^6(x+0.427411)^6}{(0.985x+0.225)^6}} - 1.2315314835401511 \times 10^{-20}) \tag{A.10}
 \end{aligned}$$

By achieving to u and d type quark distributions, all the desired combination of parton densities in the statistical model can be obtained.

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